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Concomitants of nth Upper K-record Statistics and the Current Upper K-record Statistics for Bivariate Pseudo-Weibull Distribution

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ABSTRACT

In this paper we will find the distribution of kth concomitant of nth upper k-record statistics and current upper k-record statistics for bivariate pseudo-weibull distribution. Moments properties and numerical examples and related diagrams have also been obtained for the resulting distributions.

Keywords: Concomitants, K-record value, bivariate pseudo-weibull distribution.

1. INTRODUCTION

Let $\{X_i; i \geq 1\}$ be a sequence of (iid) continuous random variables with probability density function (pdf) $f(x)$ and cumulative distribution function (cdf) $F(x)$. The k first observations in this sequence are called sample of size k. An observation kth is called an usual upper record value if its next observation value greater than of all previous observations. Now, upper k-record process is defined in terms of the kth largest X yet seen. To be more precise for positive integer k, the upper k-record times $T_n(k)$ and the upper k-record values $R_n(k)$ are defined as follows:

Let $R_{1(k)} = X_{k:k}, T_{1,k} = k$ and for $n \geq 2$

$$T_{n,k} = \min\{j : j > T_{n-1,k}, X_j > X_{T_{n-1,k}-k+1:T_{n-1,k}}\},$$

where $X_{i:n}$ denote the ith order statistic in a sample of size n. The sequence of upper k-record is then defined by

$$R_{n(k)} = X_{T_{n,k}-k+1:T_{n,k}}.$$

In the special case by putting $k=1$, one can be obtained the usual upper records (see Arnold *et al*. (1998)). Sequence of k-record was introduced by Diziubdziela and Kopocinski (1976). The pdf of nth upper k-record values of x sequence is as follows:

$$g_{n(k)}(x) = \frac{k^{n+1}}{n!}\left[-\log\{1-F(x)\}\right]^n \left[1-F(x)\right]^{k-1} f(x) \quad n \geq 0. \tag{1}$$

Further, the joint distribution of mth and nth concomitant of nth upper k-record is given by Ahsanullah (1995) as

$$\begin{aligned} g_{m,n(k)}(x_1,x_2) = {} & \frac{k^{n+1}}{m!(n-m-1)!}\left[-\log\{1-F(x_1)\}\right]^m \\ & \times\left[-\log\{1-F(x_2)\} + \log\{1-F(x_1)\}\right]^{n-m-1} \\ & \times\frac{\left[1-F(x_2)\right]^{k-1}}{1-F(x_1)} f(x_1) f(x_2) \quad x_1 < x_2. \end{aligned} \tag{2}$$

For example a rock crushing machine has to be reset if, at any operation, the size of the rock being crushed is larger than any other rock that has been crushed before. The following data, given by Ahmadi *et al*. (2011), are the sizes dealt with up to the third time that the machine has been reset:

9.3 0.6 24.4 18.1 6.6 9.0 14.3 6.6 13 2.4 5.6 33.85

The record values were the sizes at the operation when resetting was necessary. The k-record, $R_{n(k)}$, extracted from the above data set are as follows

i	1	2	3	4
$R_{i(1)}$	9.3	24.4	33.8	
$R_{i(2)}$	0.6	9.3	18.1	24.4
$R_{i(3)}$	0.6	9.3	14.3	18.1

Let $U'_{n(k)}$ is the *k*th largest random observation, when observing the *n*th *k*-record (upper) from the sequence $\{X_n; n \geq 1\}$, we call such recent statistics current *k*-records. Of course, when new observations become available, new current *k*-record can arise. In infinite sequences, every new observation that is larger than the recent upper current *k*-record will eventually become a current *k*-record.

The marginal density of $U'_{n(k)}$ is given by Houchens (1984) as

$$f_{U'_{n(k)}}(x) = 2^n f(x)\left\{1-\left(1-F(x)\right)\sum_{j=0}^{n-1}\frac{\left(-\log\{1-F(x)\}\right)^j}{j!}\right\}. \qquad (3)$$

For example, let us consider the following sequence of observations:

9.3 0.6 24.4 18.1 6.6 9.0 14.3 6.6 13 2.4 5.6 33.85

The current upper *k*-record extracted from the above sequence are as follows:

i	1	2	3	4	5	6	7
$U'_{i(1)}$	9.3	9.3	24.4	24.4	24.4	24.4	24.4
$U'_{i(2)}$	0.6	9.3	18.1	18.1	18.1	18.1	18.1
$U'_{i(3)}$	0.6	9.3	9.3	9.3	14.3	14.3	14.3

Let $\{(X_i, Y_i); i \geq 1\}$ be a sequence of independent random variables from some bivariate distribution function $F(x, y)$ and if random variables are ordered by sequence of *k*-record values in the sequence of *x's*; then the Y-variable associated with the X-value which is quantified as the k-record value is called concomitant of the k-record value.

The distribution of concomitants value introduced for first time by David (1973). So far, considerable research has not been done on concomitants of records variable. Houchens (1984) only a brief look at some of the properties, concomitant of upper record variable. He also has a short review example. Including recent work done in this case can be Ahsanullah (1994, 2000), Khaledi and Kochar (2002), Ahsanullah and Raqab (2002) and Ahsanullah *et al.* (2010) pointed out.

The distribution of concomitant of record value can be obtained by using the following expression give in Ahsanullah (1995)

$$g(y_k) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(y \mid x) g_{n(k)}(x) dx, \tag{4}$$

where $g_{n(k)}(x)$ is the *pdf* of random variable X that in this paper it will be the *pdf* of nth upper k-record value given in (1) and also nth current upper k-record value given (3). Further, the joint distribution of mth and nth concomitant is given as:

$$g_{[m,n(k)]}(y_1, y_2) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(y_1 \mid x_1) f(y_2 \mid x_2) g_{m,n(k)}(x_1, x_2) dx_1 dx_2, \tag{5}$$

where $g_{m,n(k)}(x_1, x_2)$ is given in (2).

In this paper we have obtained the distribution of concomitants of nth upper k-record statistics and also upper current k-record for bivariate pseudo-weibull distribution. The distribution and rth moment properties of concomitants and recurrence relation between moments of concomitants and also the joint distribution of concomitants and some properties on the joint moments has been studied in section 2, also distribution of concomitant upper current k-record and rth moments of concomitant are presented in section 3.

2. THE *K*-TH CONCOMITANT OF NTH UPPER *K*-RECORD VALUES AND ITS PROPERTIES

In this section the distribution and rth moment of kth concomitants of nth upper k-record statistics for bivariate pseudo-weibull distribution also the joint distribution of concomitants and some properties on the joint moments has been obtained.

The Weibull distribution has been introduced by the Weibull (1939); his article in (1951) discusses a number of applications and nowadays is the most common models used in reliability studies. This widely distributed in the branches of engineering is used to model failure times.

We define the bivariate pseudo-weibull distribution following the lines of Shahbaz and Ahmad (2009).

Suppose a random variable X has a two parameter weibull distribution with parameter β and γ_1. The density function of X is:

$$f\left(x,\beta,\gamma_1\right)=\beta\gamma_1 x^{\gamma_1-1}\exp\left(-\beta x^{\gamma_1}\right),\beta>0,\gamma_1>0,x>0. \qquad (6)$$

Now; let random variable Y has the weibull distribution with parameters $\varphi\left(x\right)$ and γ_2. The density function of Y is:

$$f\left(y,\varphi\left(x\right),\gamma_2\mid x\right)=\varphi\left(x\right)\gamma_2 y^{\gamma_2-1}\exp\left(-\varphi\left(x\right)y^{\gamma_2}\right),\varphi\left(x\right)>0,\gamma_2>0,y>0. \qquad (7)$$

On substituting $\varphi\left(x\right)=x^{\gamma_1}$ in the equation (7), the density function of the bivariate pseudo-weibull distribution is defined by

$$f(x,y)=\beta\gamma_1\gamma_2 x^{2\gamma_1-1}y^{\gamma_2-1}\exp[-x^{\gamma_1}(\beta+y^{\gamma_2})],\ \ \beta>0,\gamma_1>0,y>0,x>0. \qquad (8)$$

The conditional distribution, $f(y\mid x)$, from (8) is:

$$f(y\mid x)=\gamma_2 x^{\gamma_1}y^{\gamma_2-1}\exp\left(-x^{\gamma_1}y^{\gamma_2}\right),y>0. \qquad (8)$$

The distribution of *k*th concomitants of *n*th upper *k*-record statistics for bivariate pseudo-weibull distribution, has been obtained base on below theorem.

Theorem 1

Let $\{X_i; i \geq 1\}, i = 1, 2, \ldots$ be sequence of random variables; the distribution of kth concomitants of nth upper k-record statistics for bivariate pseudo-weibull distribution is given by

$$G(y) = 1 - \left(\frac{\beta k}{y^{\gamma_2} + \beta k} \right)^{n+1}.$$

Proof.

We have according to equation (4) by $g_{n(k)}(x)$; the probability density function nth upper k-record; and the conditional density function kth concomitants of nth upper k-record that:

$$g(y_k) = \frac{\gamma_1 \gamma_2 y^{\gamma_2 - 1} \beta^{n+1} k^{n+1}}{n!} \int_0^\infty x^{(n+2)} \gamma_1 \exp\left[x^{\gamma_1} \left(k\beta + y^{\gamma_2} \right) \right] dx.$$

Now; making the transformation $x^{\gamma_1}\left(k\beta + y^{\gamma_2}\right) = u$ we have:

$$g(y_k) = \frac{(n+1)\gamma_2 y^{\gamma_2 - 1} \beta^{n+1} k^{n+1}}{\left(k\beta + y^{\gamma_2}\right)^{n+2}}, \tag{9}$$

and the distribution function of Y is given by:

$$G(y) = 1 - \left(\frac{\beta k}{y^{\gamma_2} + \beta k} \right)^{n+1}. \tag{10}$$

Remark 2.1

The pth Percentile for construction of confidence intervals will be obtained by solving equation

$$G(y) = 1 - \left(\frac{\beta k}{y^{\gamma_2} + \beta k} \right)^{n+1} = p,$$

as from

$$y_{p-th} = \left\{ \left[\frac{(\beta k)^{n+1}}{1-p} \right]^{\frac{1}{n+1}} - \beta k \right\}^{\frac{1}{\gamma_2}}. \tag{11}$$

Remark 2.2

Taking derivative of equation (10), mode of the distribution is given as

$$Mod = \left\{ \frac{k\beta(\gamma_2 - 1)}{(n+1)\gamma_2 - 1} \right\}^{\frac{1}{\gamma_2}}. \tag{12}$$

Remark 2.3

Hazard function by using cumulative distribution function and probability density function is obtained as

$$r(x) = \frac{(n+1)\gamma_2 y^{\gamma_2 - 1}}{y^{\gamma_2} + \beta k}. \tag{13}$$

The hazard rate function behavior in the following diagrams in different models are compared.

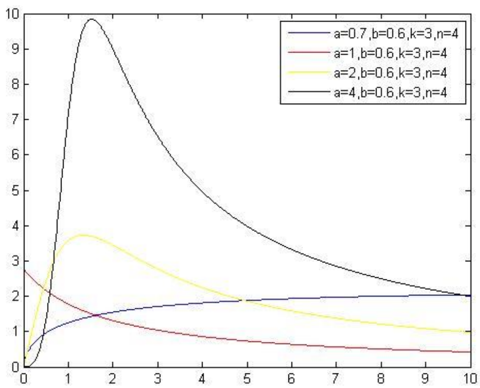

Figure 1: Hazard functions for different values of γ_2 (a= γ_2 and b= β)

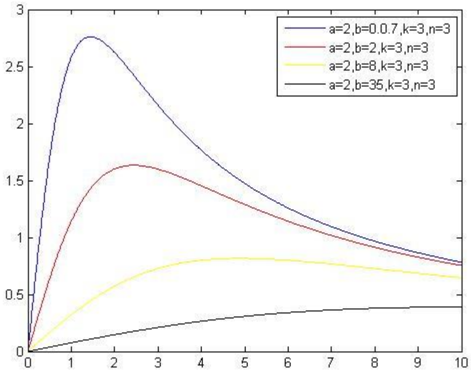

Figure 2: Hazard functions for different values of β (a= γ_2 and b= β)

In figure 1 and 2 it is showed that hazard rate function with increasing γ_2 arrives late to the attenuation and with increasing β arrives fast to the attenuation for nth upper k-record for bivariate pseudo-weibull distribution when $n = 4$ and $k = 3$.

Theorem 2

If the probability density function of kth concomitant of upper record is as follows

$$g(y_k) = \frac{(n+1)\gamma_2 y^{\gamma_2 - 1}\beta^{n+1}k^{n+1}}{\left(k\beta + y^{\gamma_2}\right)^{n+2}};$$

Then, the rth moment of the distribution is

$$\mu_r' = (k\beta)^{\frac{r}{\gamma_2}} \frac{\Gamma\left(n - \frac{r}{\gamma_2} + 1\right)\Gamma\left(\frac{r}{\gamma_2} + 1\right)}{\Gamma(n+1)}. \tag{14}$$

Proof.

The rth moment of the distribution given in (10) is obtained as

$$\begin{aligned}\mu_r' &= E\left[y^r\right] \\ &= \int_0^\infty y^r g(y_k)dy \\ &= \int_0^\infty \frac{y^r\left(n+1\right)\gamma_2 y^{\gamma_2-1}\beta^{n+1}k^{n+1}}{\left(k\beta+y^{\gamma_2}\right)^{n+2}}dy,\end{aligned}$$

making the transformation $\dfrac{\beta k}{\beta k+y^{\gamma_2}}=u,$ we have

$$\begin{aligned}\mu_r' &= \left(n+1\right)(k\beta)^{\frac{r}{\gamma_2}}\int_0^\infty u^{n-\frac{r}{\gamma_2}}\left(1-u\right)^{\frac{r}{\gamma_2}} du \\ &= \left(k\beta\right)^{\frac{r}{\gamma_2}}\frac{\Gamma\left(n-\dfrac{r}{\gamma_2}+1\right)\Gamma\left(\dfrac{r}{\gamma_2}+1\right)}{\Gamma\left(n+1\right)}.\end{aligned} \tag{16}$$

We can obtain mean and variance of the concomitant of k-record statistics for pseudo-weibull distribution with substituting $r=1, r=2$ in the above equation as follow

$$E\left[Y\right]=\left(k\beta\right)^{\frac{1}{\gamma_2}}\frac{\Gamma\left(n-\dfrac{1}{\gamma_2}+1\right)\Gamma\left(\dfrac{1}{\gamma_2}+1\right)}{\Gamma\left(n+1\right)},$$

$$Var\left(Y\right)=\frac{(k\beta)^{\frac{2}{\gamma_2}}}{n!\gamma_2^{\,2}}\left[\left(2n\gamma_2-4\right)\Gamma\left(n-\frac{2}{\gamma_2}\right)\Gamma\left(\frac{2}{\gamma_2}\right)-\frac{\left(n-\dfrac{1}{\gamma_2}\right)^2\Gamma^2\left(n-\dfrac{1}{\gamma_2}\right)\Gamma^2\left(\dfrac{1}{\gamma_2}\right)}{n!}\right].$$

The table 1 and 2 show numerical values of mean and variance of the concomitant of nth upper k-record statistics for pseudo-weibull distribution for $k=3, n=4$ and different value of β and γ_2 .

TABLE 1: Mean of the concomitant of nth upper k-record statistics for pseudo-weibull distribution for $k = 3, n = 4$

β | γ2	0.7	0.8	1	1.2	1.4	1.6
0.6	0.4395	0.4353	0.4500	0.4750	0.5021	0.5285
0.7	0.5478	0.5278	0.5250	0.5401	0.5605	0.5819
0.9	0.7844	0.7227	0.6750	0.6660	0.6707	0.6809
1	0.9119	0.8244	0.7500	0.7271	0.7231	0.7272

In table 1 we see the mean of the concomitant of nth upper k-record statistics for pseudo-weibull distribution in first reduce and then increases with increasing γ_2 and mean increases with increasing β.

TABLE 2: Variance of the concomitant of nth upper k-record statistics for pseudo-weibull distribution for $k = 3, n = 4$

β | γ2	0.7	0.8	1	1.2	1.4	1.6
0.6	1.0081	0.6107	0.3375	0.2383	0.1875	0.1561
0.7	1.5659	0.8978	0.4595	0.3080	0.2336	0.1892
0.9	3.2108	1.6828	0.7594	0.4683	0.3346	0.2591
1	4.3386	2.1899	0.9375	0.5582	0.3889	0.2955

In table 2 we see the variance of the concomitant of nth upper k-record statistics for pseudo-weibull distribution decreases with increasing γ_2 and variance increases with increasing β.

Based on relations between the *pdf* and hazard function as

$$\overline{F}(y) = g(y_k)\left(\frac{k\beta + y^{\gamma_2}}{(n+1)\gamma_2 y^{\gamma_2-1}}\right), \tag{15}$$

where $\overline{F}(y)=1-F(y);\mathbb{S}$ and using (17) in (16) by simplifying, we obtain following recurrence relation for moments of concomitants of record statistics:

$$\mu_r = \frac{rk\beta}{(n+1)\gamma_2 - r}\mu_{r-\gamma_2}.$$

In this section we have also obtained the joint distribution of *m*th and *n*th concomitant of *n*th upper *k*-record statistics. For this reason; consider the joint distribution of *m*th and *n*th concomitant given in (5). To obtain the distribution, we first obtain the joint distribution of *m*th and *n*th concomitant of nth upper *k*-record statistics for random variable *X* by using (2). This distribution is

$$g_{m,n(k)}\left(x_1,x_2\right)=\frac{(k\beta)^{n+1}\gamma_1^{\,2}}{m!(n-m-1)!}\left[x_2^{\gamma_1}-x_1^{\gamma_1}\right]^{n-m-1}\exp\left(-k\beta x_2^{\gamma_1}\right)x_1^{(m+1)\gamma_1-1}x_2^{\gamma_1-1}. \tag{16}$$

Now using (18) and (9) in (5) and after some calculus, the joint distribution of two concomitants of *k*-records is:

$$g\left(y_1,y_2\right)=\frac{(k\beta)^{n+1}\gamma_1^{\,2}\gamma_2^{\,2}y_1^{\gamma_2-1}y_2^{\gamma_2-1}}{m!(n-m-1)!}\int_0^{\infty}x_2^{2\gamma_1-1}\gamma_1\exp\left[-x_2^{\gamma_1}\left(k\beta+y^{\gamma_2}\right)\right]dx_2$$

$$\times\underbrace{\int_0^{x_2}\left[x_2^{\gamma_1}-x_1^{\gamma_1}\right]^{n-m-1}x_1^{(m+2)\gamma_1-1}\exp\left(-x_1^{\gamma_1}y_1^{\gamma_2}\right)dx_1}_{A(x)},$$

making the transformation $\left(\dfrac{x_1}{x_2}\right)^{\gamma_1}=u$ in $A(x)$, we have

$$A(x)=\frac{x_2^{(n+1)\gamma_1}\Gamma(m+2)\Gamma(n-m)}{\gamma_1\Gamma(n+2)}\,{}_1F_1\left(m+2,n+2,-x_2^{\gamma_1}y_1^{\gamma_2}\right).$$

Now

$$g\left(y_1,y_2\right)=\frac{(k\beta)^{n+1}\gamma_1^{\,2}\gamma_2^{\,2}y_1^{\gamma_2-1}y_2^{\gamma_2-1}}{m!(n-m-1)!}\frac{(m+1)\Gamma(m+1)\Gamma(n-m)}{\gamma_1\Gamma(n+2)}$$

$$\times\int_0^{\infty}x_2^{(n+3)\gamma_1-1}\exp\left[-x_2^{\gamma_1}\left(k\beta+y^{\gamma_2}\right)\right]{}_1F_1\left(m+2,n+2,-x_2^{\gamma_1}y_1^{\gamma_2}\right)dx_2,$$

which, on further simplification, provide following distribution of two concomitants of records

$$g(y_1,y_2)=(k\beta)^{n+1}\gamma_2{}^2 y_1{}^{\gamma_2-1}y_2{}^{\gamma_2-1}\left(k\beta+y_2{}^{\gamma_2}\right)^{m-n-1}\left[\frac{(n+2)(m+1)}{\left(y_1{}^{\gamma_2}+y_2{}^{\gamma_2}+k\beta\right)^{m+2}}\right.$$
$$\left.-\frac{(m+1)(m+2)\,y_1{}^{\gamma_2}}{\left(y_1{}^{\gamma_2}+y_2{}^{\gamma_2}+k\beta\right)^{m+3}}\right], \qquad (17)$$

where $Y_1=Y_m, Y_2=Y_n$.

The product moments of mth and nth concomitants can be obtained by using:

$$\mu_{q,r}=E\left[Y^qY^r\right]$$
$$=\int_0^\infty\int_0^\infty y_1{}^q y_2{}^r g(y_1,y_2)dy_1dy_2$$

$$=\left\{\frac{\Gamma\left(\frac{q}{\gamma_2}+1\right)\Gamma\left(\frac{r}{\gamma_2}+1\right)\Gamma\left(m-\frac{q}{\gamma_2}+1\right)\Gamma\left(n-\frac{q}{\gamma_2}-\frac{r}{\gamma_2}+1\right)}{(k\beta)^{-\frac{q+r}{\gamma_2}}\Gamma(m+1)\Gamma\left(n-\frac{q}{\gamma_2}+2\right)}\right\}\left[n-\frac{q}{\gamma_2}+1\right]. \quad (18)$$

Equation (20) can be used to find covariance and correlation coefficient between concomitants.

3. DISTRIBUTION AND MOMENT OF K-TH CONCOMITANT OF UPPER CURRENT *K*-RECORD VALUES

Let $\{X_i; i\ge 1\}$ be a sequence of *iid* continuous random variables. To get the first current k-record, the first partial sample of size k, $\{X_1,\cdots,X_k\}$, from the above sequence is needed. Suppose $U'_{1,k}$ the kth largest (smallest) observation among them is defined as the first upper (lower) current k-record. That is $U'_{1,k}=X_{1,k}$. if $\{X_{k+1}>U'_{1,k}\}$ then X_{k+1} creates the second current k-record. The afore mentioned event occurs with

probability 1 and therefore X_{k+1} certainly induces the second current *k*-record. Formally, we have $U'_{2,k} = X_{2:k+1}$. Therefore the second current *k*-record arises by adding only one observation to the first partial sample of size k. The *n*th current *k*-record arises in the same way when n $\leq$ k + 1.
In this section we have derived the distribution of *k*th concomitants of current upper *k*-record statistics for pseudo-weibull distribution.
The distribution of current upper *k*-record statistics for pseudo-weibull distribution is given as

$$f_{U'_n}(x) = 2^n\left(\beta\gamma_1 x^{\gamma_1-1}\exp(\beta x^{\gamma_1})\right)\left\{1-\sum_{j=0}^{n-1}\exp(\beta x^{\gamma_1})\frac{(\beta x^{\gamma_1})^j}{j!}\right\}, x>0, \qquad (19)$$

Theorem 3

Let $\{X_i; i \geq 1\}, i = 1, 2, \ldots$ be sequence of random variables; the distribution function of *k*th concomitants of upper current *k*-record statistics for bivariate pseudo-weibull distribution is given by

$$F(y) = 2^n\left[1-\frac{\beta}{\beta+y^{\gamma_2}}\right]+\sum_{j=0}^{n-1}2^n\left[\left(\frac{\beta}{2\beta+y^{\gamma_2}}\right)^{j+1}-\frac{1}{2^{j+1}}\right].$$

Proof.

By using (9) and (21) in (4), we get

$$h(y) = \int_0^\infty f(y\,|\,x) f_{U'_n}(x)dx$$

$$= \int_0^\infty \gamma_1 x^{\gamma_1} y^{\gamma_2-1}\exp(-x^{\gamma_1}y^{\gamma_2})\left(2^n\beta\gamma_1 x^{\gamma_1-1}\exp(\beta x^{\gamma_1})\right)\left\{1-\sum_{j=0}^{n-1}\exp(\beta x^{\gamma_1})\frac{(\beta x^{\gamma_1})^j}{j!}\right\}dx,$$

making the transformation $x^{\gamma_1}\left(\beta + y^{\gamma_2}\right) = u, \quad x^{\gamma_1}\left(2\beta + y^{\gamma_2}\right) = w,$

we have

$$h(y)=\beta 2^{n}\gamma_{2}y^{\gamma_{2}-1}\left[\frac{1}{\left(\beta+y^{\gamma_{2}}\right)^{2}}-\sum_{j=0}^{n-1}\frac{(j+1)\beta^{j}}{\left(2\beta+y^{\gamma_{2}}\right)^{j+2}}\right]. \qquad (22)$$

The density function of Y , distribution function of Y is given by

$$F(t)=\int_{0}^{y}h(t)dt$$

$$=\int_{0}^{y}\left[\frac{\beta 2^{n}\gamma_{2}t^{\gamma_{2}-1}}{\left(\beta+y^{\gamma_{2}}\right)^{2}}-\beta 2^{n}\gamma_{2}t^{\gamma_{2}-1}\sum_{j=0}^{n-1}\frac{(j+1)\beta^{j}}{\left(2\beta+y^{\gamma_{2}}\right)^{j+2}}\right]dt$$

$$=2^{n}\left[1-\frac{\beta}{\beta+y^{\gamma_{2}}}\right]-\sum_{j=0}^{n-1}2^{n}\left[\frac{1}{2^{j+1}}-\left(\frac{\beta}{2\beta+y^{\gamma_{2}}}\right)^{j+1}\right]. \qquad (20)$$

Also by using (22) and (23), the hazard rate function of the distribution is

$$r(x)=\frac{\beta 2^{n}\gamma_{2}y^{\gamma_{2}-1}\left[\frac{1}{\left(\beta+y^{\gamma_{2}}\right)^{2}}-\sum_{j=0}^{n-1}\frac{(j+1)\beta^{j}}{\left(2\beta+y^{\gamma_{2}}\right)^{j+2}}\right]}{1-2^{n}\left[1-\frac{\beta}{\beta+y^{\gamma_{2}}}\right]-\sum_{j=0}^{n-1}2^{n}\left[\frac{1}{2^{j+1}}-\left(\frac{\beta}{2\beta+y^{\gamma_{2}}}\right)^{j+1}\right]}.$$

In figure 3 we shown hazard rate function with increasing γ_2 and constant β arrives fast to the attenuation and In Figure 4 with increasing β and constant γ_2 arrives late to the attenuation for upper current *k*-record for bivariate pseudo-weibull distribution when $n=4$ and $k=3$.

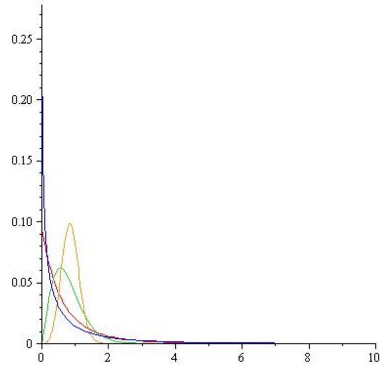

Figure 3: Hazard functions for different values of γ_2

Now, the *r*th moment of the *k*th concomitants of upper current *k*-record statistics for bivariate pseudo-weibull distribution, has been obtained by using (22) as

$$\mu_r' = E\left[y^r\right]$$

$$= \int_0^\infty h(y)dy$$

$$= \int_0^\infty \frac{y^r \beta 2^n \gamma_2 y^{\gamma_2 - 1}}{\left(\beta + y^{\gamma_2}\right)^2} dy - \int_0^\infty \sum_{j=0}^{n-1} y^r \beta 2^n \gamma_2 y^{\gamma_2 - 1} \frac{\left(j+1\right)\beta^j}{\left(2\beta + y^{\gamma_2}\right)^{j+2}} dy, \quad (21)$$

making the transformations $\frac{\beta}{2\beta + y^{\gamma_2}} = w$ and $\frac{\beta}{\beta + y^{\gamma_2}} = u,$ we have

$$\mu_r' = 2^n \beta^{\frac{r}{\gamma_2}} \Gamma\left(1 + \frac{r}{\gamma_2}\right)\left[\Gamma\left(1 - \frac{r}{\gamma_2}\right) - \sum_{j=0}^{n-1} \frac{\Gamma\left(j - \frac{r}{\gamma_2} + 1\right)}{2^{j - \frac{r}{\gamma_2} + 1} j!}\right] \quad (22)$$

The mean and variance of the distribution with substituting $r = 1, r = 2$ in above equation are

$$E(Y) = 2^n \beta^{\frac{1}{\gamma_2}} \Gamma\left(1+\frac{1}{\gamma_2}\right)\left[\Gamma\left(1-\frac{1}{\gamma_2}\right) - \sum_{j=0}^{n-1} \frac{\Gamma\left(j-\frac{1}{\gamma_2}+1\right)}{2^{j-\frac{1}{\gamma_2}+1} j!}\right], \quad \gamma_2 \neq 1,$$

$$Var(Y) = 2^n \beta^{\frac{2}{\gamma_2}} \left\{\Gamma\left(1+\frac{2}{\gamma_2}\right)\left[\Gamma\left(1-\frac{2}{\gamma_2}\right) - \sum_{j=0}^{n-1} \frac{\Gamma\left(j-\frac{2}{\gamma_2}+1\right)}{2^{j-\frac{2}{\gamma_2}+1} j!}\right]\right.$$

$$\left. -2^n \Gamma^2\left(1+\frac{1}{\gamma_2}\right)\left[\Gamma\left(1-\frac{1}{\gamma_2}\right) - \sum_{j=0}^{n-1} \frac{\Gamma\left(j-\frac{1}{\gamma_2}+1\right)}{2^{j-\frac{1}{\gamma_2}+1} j!}\right]^2\right\}, \quad \gamma_2 \neq 1.$$

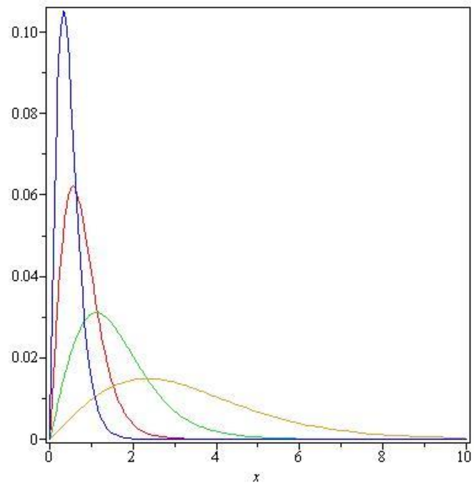

Figure 4: Hazard functions for different values of β

In Tables 3 and 4, we presented some different value of mean and variance.

TABLE 3: Mean of the concomitant of current upper k-record statistics for $k = 3$, $n = 4$

β | γ2	0.7	0.8	1.2	1.4	1.6
0.6	0.1954	0.2138	0.2951	0.3333	0.3694
0.7	0.2435	0.2592	0.3356	0.3720	0.4068
0.9	0.3487	0.3549	0.4138	0.4452	0.4759
1	0.4053	0.4049	0.4518	0.4800	0.5083

In table 3 we see that mean of the concomitant of current upper k-record statistics for pseudo-weibull distribution increases with increasing β and γ_2.

TABLE 4: Variance of the concomitant of current upper k-record statistics for $k = 3, n = 4$

β | γ2	0.7	0.8	1.2	1.4	1.6
0.6	0.2054	0.1511	0.0937	0.0834	0.0773
	0.3190	0.2221	0.1211	0.10510	0.0938
0.9	0.6543	0.4163	0.1841	0.1505	0.1248
1	0.8840	0.5418	0.2194	0.1749	0.1464

In table 4 we shown variance of the concomitant of current upper k-record statistics for pseudo- weibull distribution increases with increasing β and variance decreases with increasing γ_2.

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REFERENCES

Ahmadi, J., MirMostafaa, M.T.K. and Balakrishnan, N. (2011). Bayesian prediction of order statistics Based on k-record value from exponential distribution. *Statistics*. **45**(4): 375-387.

Ahsanullah, M. (1994). Record Values, random models and concomitants. *J. Statist. Res.* **28**: 89-109.

Ahsanullah, M. (1995). *Record statistics*. Nova Science Publishers, USA.

Ahsanullah, M. (2000). Concomitants of record values. *Pakistan J. Statist*. **16**: 207-215.

Ahsanullah, M., Sahbaz, S. Sahbaz, M. Q. and Mohsin, M. (2010). Concomitant of upper records statistics for bivariate psudo-weibull distribution. *Application and Applied Mathematics*. **5**: 1379-1388.

Arnold, B. C., Balakrishnan, N. and Nagaraja, H. N. (1998). *Records.* New York : Wilely.

David, H. A. and Nagaraja, H. N. (2003). Order statistics. New York: Wiley.

Dziubdziela, W. and Kopocinski, B. (1976). Limiting properties of k-th record values. *Appl. Math.* **15**: 187-190.

Houchens, R. L. (1984). *Record value theory and inference*. PhD Dissertation, University of California, Riverside, C. A.

Khaledi, B. and Kochar, S. C. (2001). On dependence structure of multivariate mixture distributions. *Ann. Inst. Stat. Math.* **53**: 620-630.

Raqab, M. Z. and Ahsanullah, M. (2002). Concomitants of ordered random variables a review. *J. Stat. Theory App.* **1**: 15-26.

Shahbaz, S. and Ahmad, S . (2009). Concomitants of order statistics for bivariate pseudo-weibull distribution. *World App. Sci. J.* **6** (10): 1409-1412.

Weibull, W. (1939b). *The phenomenon of rupture in solids.* Reports No. 153. Inggeniors Vetenscaps Akademines Handligar, Stokhoms.

Weibull, W. (1951). A statistical distribution of wide applicability. *Journal of Applied Mechanics.* **18**: 293-297.